MAT 1341 C, Quiz 1

Solution

February 4, 2019

Length: 15 minutes.

Professor: Rachid Bentoumi.

Family name:____________________

First name:____________________

Student number:____________________

1	A
2	E
3	B
Total	

PLEASE CAREFULLY READ THESE INSTRUCTIONS:

1. Carefully read each question and **record your responses in the space provided on this page as well as the question page.**
2. You are not allowed to consult your notes or any books. Calculators, phones, and other electronic devices are not allowed.
3. There are three multiple choice questions, each worth 1 point. No partial credit will be awarded. **You must indicate the method you used to select the correct answer; unjustified answers will not be given credit.**

   Record your answers both on the question page and on the title page.

1. Which of the following subsets are subspaces of $M_2(\mathbb{R})$?

✓ A. $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a+d=0\right\}$

✗ B. $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} ; ad=1\right\}$

✗ C. $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \text{ are integers}\right\}$

✗ D. $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} ; ad-bc=0\right\}$

✗ E. $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a=1\right\}$

✗ F. None of these subsets are subspaces.

Solution (**A) $A = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} ; d=-a\right\} = \left\{\begin{bmatrix} a & b \\ c & -a \end{bmatrix} \middle| a, c, b \in \mathbb{R}\right\}$

* $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in A$ for $a=b=c=d=0$

* for $u \in A \Rightarrow u = \begin{bmatrix} a_1 & b_1 \\ c_1 & -a_1 \end{bmatrix}$ and for $v \in A \Rightarrow v = \begin{bmatrix} a_2 & b_2 \\ c_2 & -a_2 \end{bmatrix}$

Now, $u+v = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & -(a_1+a_2) \end{bmatrix} \in A$

* for $u \in A \Rightarrow u = \begin{bmatrix} a_1 & b_1 \\ c_1 & -a_1 \end{bmatrix}$ and $K \in \mathbb{R}$ $Ku = K\begin{bmatrix} a_1 & b_1 \\ c_1 & -a_1 \end{bmatrix} =$

$\begin{bmatrix} Ka_1 & Kb_1 \\ Kc_1 & -Ka_1 \end{bmatrix} \in A$. Hence A is a subspace of $M_2(\mathbb{R})$.

(**) $B = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} ; ad=1\right\}$. since $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin B \Rightarrow B$ is not a subspace of $M_2(\mathbb{R})$. (Take $a=0, b=0, c=0, d=0, ad=0 \neq 1$)

(**) C is not closed under scalar multiplication. For example $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix}$, but $a=b=c=d=\frac{1}{3}$ are not integers

(**) D is not closed under addition: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \in D$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \in D$ but $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \notin D$ (since $(1)(2)-(1)(1) \neq 0$)

(**) E is not a subspace of $M_2(\mathbb{R})$ since

$E = \left\{ \begin{bmatrix} 1 & b \\ c & d \end{bmatrix},\ b,c,d \in \mathbb{R} \right\}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin E$

2. Let $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 0\}$. Which of the following subsets of $\mathbb{R}^3$ are a spanning set for W?

A. $\{(0, 0, 0)\}$

B. $\{(1, 1, -1)\}$

C. $\{(-1, 1, 0), (1, -1, 0)\}$

D. $\{(1, 0, 1)\}$

E. $\{(-1, 1, 0), (1, 0, 1)\}$

F. $\{(1, 1, -1), (-1, 1, 0)\}$

Solution:

$$W = \{(x,y,z) \in \mathbb{R}^3 \mid x = -y+z\} = \{(-y+z, y, z) \in \mathbb{R}^3 \mid y, z \in \mathbb{R}\}$$

$$= \{(-y, y, 0) + (z, 0, z) \mid y, z \in \mathbb{R}\}$$

$$= \{y(-1,1,0) + z(1,0,1) \mid y, z \in \mathbb{R}\}$$

$$= \text{span}\{(-1,1,0), (1,0,1)\}$$

3. Let $\mathbb{P}_2 = \{\, p \mid p(x) = a + bx + cx^2, \text{ where } a, b, c \in \mathbb{R}\}$, the vector space consisting of polynomials of degree less than or equal to 2 with real coefficients.

Consider the following subset of $\mathbb{P}_2$:

$$S = \{x^2 - 1,\ x^2 + 1,\ x - 1,\ x + 1\}$$

Which of the following statements about S is true?

I. S is linearly dependent
II. S is linearly independent
III. S spans $\mathbb{P}_2$
IV. S is a basis of $\mathbb{P}_2$

A. (I) and (II)
B. (I) and (III)
C. (II) and (IV)
D. (II) and (III)
E. (I), (III) and (IV)
F. (III) and (IV)

See solution in quiz1 Practice